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Nonlinear behaviour of interacting oscillating water columns

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Abstract

A preliminary study is made of the dynamics of Oscillating Water Columns (OWCs), in which the amplitude of internal fluid displacement is nonlinear. The OWC is a well-studied class of ocean wave-energy converters. When the machine's natural frequency is appropriately tuned, it will resonate at the prevailing ocean-swell frequency. Large amplitudes at resonance are desirable since they maximise the power available for electricity generation; however, large amplitudes also imply nonlinear behaviour. Most analyses of wave-energy converters of any class assume linear dynamics. Furthermore, multiple machines are now constructed in close proximity to maximise economic returns, and hence they interact. The OWC is reduced to a single degree-of-freedom nonlinear oscillator based on appropriate simplifications of the Navier-Stokes equations. Nonlinear machines were then coupled together by inter-machine wave scattering. It is found that provided wave amplitudes are realistic for Southern Ocean conditions, linear theory remained reasonable for both single and coupled machines. However, nonlinearities could significantly reduce resonant power output.

Introduction

Wave Energy Converters (WEC) are machines that extract renewable energy from ocean waves. Although the first patent for a WEC was filed in 1799 [10], the reciprocating (constantly reversing) flow created by wave action, plus the need to have the designs of large machines in the ocean as simple as possible, has meant that there is no obvious method of efficiently extracting power from ocean waves. Consequently, they are at present over 250 companies with competing designs of WECs [5]. Despite this apparent diversity, the majority of designs rely on the principle of resonance: the machine is designed to have a natural frequency with which it oscillates in response to a disturbance. When the natural frequency is similar to the frequency of prevailing ocean swell, resonance occurs. The resonating machine oscillates with an amplitude greater than that of the incoming waves; this represents power extracted from an area of the wave field larger than the machine's physical size.

Some concepts are now being installed in arrays or 'farms' to achieve economies of scale. When the machine is a resonator, it has the ability to interact significantly with its neighbours. Indeed, early theories on WEC array interactions predicted that a correctly-spaced array would deliver approximately 50% more power than the same number of isolated machines, while an incorrectly-spaced array would deliver approximately 50% less [4]. Interaction occurs both via the diffraction of waves, if the machine size is significant compared to the wavelength, and also by the radiation of waves that represents the resonance, as noted below. A substantial body of research has investigated array interactions, e.g. [8, 11, 12, 6, 19, 7]. However, the majority of studies assume the behaviour of the machine is governed by linear dynamics, and that the waves it radiates are linear.

For a machine that is much smaller than the wavelength, the radiated wave field is described by Hankel functions, which reduce rapidly in amplitude with distance from the origin. Thus, even if the machine at the centre operates nonlinearly, it is reasonable to continue to assume the radiated waves coupling it to its neighbours are linear. Meanwhile, linearity of the diffracted waves is retained provided the incoming swell is linear. Indeed, recent studies suggest that nonlinearity in the coupling wave fields has only a small influence on the array dynamics [22].

However, the intent of the WEC design is for the machine itself to resonate, and thus, by definition, reach a large amplitude to maximise power extraction. Thus, the question is raised of whether the dynamics of each individual machine needs to be modelled as a nonlinear oscillator. Once the linear versus nonlinear dynamics of the individual machine is determined, the machine may be coupled to its neighbours via coupling terms which may be justifiably linear as noted above. In the present paper, the dynamics of the oldest-established class of WEC, the Oscillating Water Column (OWC) is investigated. To what extent can its behaviour be considered linear?

Formulation of individual-machine dynamics

As it oscillates, a WEC radiates waves of its own. In the potential-flow paradigm conventionally used to model WECs, e.g. [9, 8], the superposition of these radiated waves with the incoming ocean swell represents a reduction in the net amplitude of the ocean swell, and thus power extraction. Of course, potential flow is conservative, so the useful power extracted from the machine must be prescribed by a separate dissipative term. Meanwhile, a component of the wave radiation represents power that is inevitably lost from the machine, because the machine's motion does not perfectly couple with the incoming wave field. Likewise, this 'radiation damping' represents a dissipative loss in the machine dynamics. The matching of dissipative fluid dynamics (quarantined within an imaginary boundary) with the conservative potential-flow fields outside is an interesting mathematical problem which may have parallels in allied topics, such as ice-floe interactions with ocean waves e.g. [18].

The linear damping ratio for the machine dynamics, ζ, can be considered to be composed of two terms, such that $\zeta = \zeta_\mu + \zeta_P$, where ζ_μ represents all linear dissipative losses, conventionally assumed to be dominated by wave radiation, and ζ_P represents the useful power extracted. It is easy to show that for any linear, damped oscillator driven at resonance, the useful power extracted is maximised when the controllable damping due to power extraction is equal to the uncontrollable damping due to all other losses [2, 10], so that $\zeta_\mu = \zeta_P$. An immediate implication is that a linear WEC can at most convert 50% of the available power from ocean waves; the remainder must be lost to dissipation. Below, we keep ζ as a combined and constant parameter, which assumes that an operator will always seek to draw the maximum useful power. However, it should be noted that in general, the component of linear damping due to radiation depends on frequency [9].

The OWC in its simplest form is a tube or duct open at both ends, with the bottom opening of the duct immersed such that a length L is underwater. The top of the duct is in the air, and is fitted with a turbine designed to rotate in the same direction in both the upwards and downwards motion of air driven by the rising and falling water column [10]. An elementary consideration of the momentum balance of this system shows that when the water inside the tube is displaced infinitesimally from rest, in a 'piston-like' motion such that the free surface remains flat, it is a oscillator with natural frequency $\omega_0 = \sqrt{g/L}$ where g is the acceleration due to gravity. Effectively, the water contained in the duct behaves as a liquid pendulum. Various versions of OWCs have been powered by ocean waves since 1885 and a number have been grid-connected [16].

Integration of the Navier-Stokes equations over the duct cross-section and along the submerged duct length, as in [17], reduces the momentum balance to an ordinary differential equation in the vertical displacement of the free surface. Introducing the scalings $\zeta^{-1}F$ for length, where F is the amplitude of incoming waves, and ω_0^{-1} for time, leads to a momentum balance in the non-dimensional vertical displacement of the free surface, δ,

$$(1+\varepsilon\delta)\ddot{\delta}+2\zeta\dot{\delta}+\varepsilon C_f|\dot{\delta}|^{3/4}\dot{\delta}+\delta=\zeta e^{i\alpha t}, \tag{1}$$

where $\alpha = \omega/\omega_0$, with ω being the frequency of the incoming ocean waves forcing the machine, $\varepsilon = F/(\zeta L)$, and C_f is the nonlinear damping co-efficient.

The parameter ε is small for forcing amplitudes F that are very small relative to the submerged length L. Including ζ in the scaling is convenient for analytic approximations of the nonlinear behaviour, since it permits convergence of such approximations near the linear resonance. The component of the damping ratio due to linear radiation damping and laminar friction, ζ_μ, can estimated from various theoretical analyses of OWCs (e.g. [9, 15]) to be roughly 0.03-0.05 in deep water at resonance. Other linear frictional losses would inevitably affect the machine, adding to ζ_μ. Thus, as noted above, the overall value of ζ including the equal damping ζ_P due to the useful power drawn (the 'power take-off') would be roughly 0.1. Hence, ε as defined in the present paper will remain less than unity for forcing amplitudes less than roughly a tenth of the submerged machine length. Under these circumstances ε may be useful in analytic approximations that may aid understanding, as described below. However, in the present paper, we also simply integrate (1) in time to determine a nonlinear response for arbitrary values of ε. Recalling that the present work ignores the frequency-dependence of ζ_μ, the present results should be considered relevant close to the linear resonance.

There are two nonlinear terms in (1), a nonlinear inertia, $\varepsilon\delta\ddot{\delta}$, which arises from the mass of water in the column varying with displacement, and a nonlinear dissipation, $\varepsilon C_f|\dot{\delta}|^{3/4}\dot{\delta}$. Regarding C_f, experiments have shown that the flow in such systems can be laminar at zero velocity, then transition to turbulence during the deceleration phase, then re-laminarise [1]. It is still unclear what damping models to use in order to correctly capture the possibility of periodic re-laminarisation of an otherwise turbulent flow [14]; the laminar fraction of the oscillation cycle can be 50% [1]. The maritime engineering literature has established empirical correlations for the drag on structures due to ocean waves, such as the Morison equation, e.g. [20], while other models are appropriate for reciprocating flow in ducts and tubes. Consideration of the available models in the literature by [15] suggests the best model for C_f is given by

$$C_f = 0.15816\,\mathrm{Re}_\omega{}^{-1/4}, \tag{2}$$

where the oscillatory or 'kinetic' Reynolds number is given by $\mathrm{Re}_\omega = \omega D^2/\nu$, with D being the horizontal length-scale of the duct and ν the kinematic viscosity. This assumes the 'worst case' scenario where the full-scale flow is turbulent throughout the oscillation cycle. In any case, we will shortly see that even in the worst-case scenario, the nonlinearity owing to turbulent damping has a small influence relative to the nonlinearity due to inertia. However, (2) accounts only for turbulent wall friction and not for vortex formation at the OWC entrance, which will need to be considered in a future study.

It is possible to estimate some of the effects of the nonlinearities provided ε is small, by expanding δ as

$$\delta = \delta_0+\varepsilon\delta_1+\varepsilon^2\delta_2+\dots\,, \tag{3}$$

where δ_0 is the leading-order (linear) solution, δ_1 is the $O(\varepsilon)$ solution, and so on. At $O(\varepsilon)$, the variable δ_1 appears in a linear oscillator equation forced by the two nonlinear terms already known from the leading-order solution, $\delta_0\ddot{\delta}_0$ and $C_f|\dot{\delta}_0|^{3/4}\dot{\delta}_0$. However, since $C_f \ll 1$ for any realistic conditions, the nonlinear inertia term dominates. Since $\delta_0\ddot{\delta}_0$ varies as $\cos^2(\alpha t)$, it has a time dependence of $2\alpha t$ and thus will produce a nonlinear resonance in the δ_1 solution at $\alpha = 0.5$.

Method

Integration of (1) was undertaken for a range of dimensionless forcing frequencies α. The parameter ε was allowed to take values greater than unity, so that asymptotic expansions such as (3) would no longer be valid. The initial displacement and velocity were set at zero; variations from these initial conditions affected the time taken for transients to die out, but did not affect the final results. The intent was to synthesise a version of the classical linear response function, but under the nonlinear dynamics. Therefore, peaks and troughs in the time series were identified, and the nonlinear 'amplitude' of response, X, was defined as half the difference between a peak and the preceding trough. The integration was run until X converged to within 10^{-4} to ensure transients had died out, which typically took the order of ten forcing cycles or less. However, around $\alpha = 2$ for $\varepsilon > 1$, a small subharmonic at $\alpha/2$ was a persistent feature of the forced response, which meant that every second value of X differed by about 10^{-2}, preventing convergence between $\alpha = 2.0$ and 2.1. To overcome this, X was averaged over two cycles, which had no effect on the response elsewhere.

The largest value of ε tested was 1.69; above this value, bifurcations in the nonlinear dynamics introduce further high-amplitude frequencies into the timeseries that make the simplistic method of measuring peaks and interpreting them as a response function inappropriate. Consider what the value of $\varepsilon = 1.69$ translates to in practical terms. On the coasts of Australia exposed to Southern Ocean swell, the wave frequency is typically below 0.1 Hz [3], so ω is less than about 0.6 rad s^{-1}. To resonate at this frequency requires a length L of about 25 m, and indeed an OWC constructed for these conditions in 2014 had roughly this internal water-column length. Hence, if $\varepsilon = 1$ for $\zeta = 0.1$, forcing wave amplitudes should be 2.5 m – a wave height of 5 m. The average significant wave height in Perth is approximately 3-4 m [3]. Meanwhile, $\varepsilon = 1.69$ corresponds to waves over 8 m, which are beyond the range expected for normal operations in any Australian waters at any time of year [3]. Although ζ may be lower away from resonance owing to the frequency-dependence of ζ_μ, the largest amplitudes remain those at resonance. Thus, the largest nonlinearity explored in the present paper represents roughly twice the wave heights likely to be relevant in practice.

Nonlinear single machine results

Even for the extreme value of $\varepsilon = 1.69$, the dissipative nonlinearity has only a small influence on the response amplitude, and only at resonance. The peak is reduced by about 9%, con-

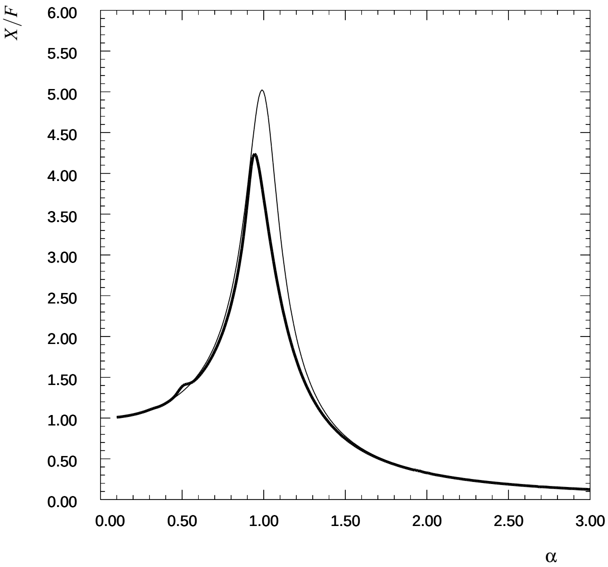

Figure 1: Amplitudes of nonlinear response X/F of water inside a single OWC (thick line) as a function of dimensionless forcing frequency α, considering both inertia-term nonlinearity and turbulent-dissipation nonlinearity. Thin line: classical response function for a linear oscillator. Damping ratio $\zeta_\mu = 0.1$; ε has the extreme value of 1.69.

sistently with other findings [15] and as predictable from the asymptotic considerations above.

Including both the nonlinear inertia and the nonlinear damping terms gives the response shown in figure 1. The resonant peak is reduced by about 15% and has shifted to a lower frequency of $\alpha = 0.94$. There is also a small peak at $\alpha = 0.5$, consistent with the nonlinear resonance at $O(\varepsilon)$ expected from the asymptotic considerations. The reduction in resonant peak height scales with ε, and thus is approximately proportional to F/L. At F/L of 0.0846 ($\varepsilon = 0.846$ for $\zeta = 0.1$), there is a reduction in resonant peak height of about 7%. For the Southern Ocean, $\varepsilon = 0.846$ corresponds to wave heights of about 4 m, which are high but realistic [3]. Even at $\varepsilon = 1.69$, timeseries display only slight variations from sinusoidal behaviour, and at $\varepsilon = 0.846$ the behaviour is closer still to sinusoidal.

Nonlinear coupled-oscillator model and results

To examine the influence of realistic Southern Ocean conditions ($\varepsilon = 0.846$) on the nonlinear dynamics of interacting machines, a pair of machines was modelled by a set of equations derived from (1). Two issues must be considered: the derivation of the assumed-linear coupling between the machines; and the treatment of multiple scattering phenomena. In multiple scattering, there is an infinite series of wave reflexions linking the machines, requiring an awkward, infinitely nested set of sums [13].

To derive the coupling terms representing the interactions, consider the surface-wave velocity potential around WECs, ϕ. This is usually represented as the sum of three components (e.g. [4]),

$$\phi = \phi_i + \phi_d + \phi_r, \tag{4}$$

where the terms on the right-hand side are the incident, diffracted and radiated potentials respectively. The potential ϕ_r is a solution to Laplace's equation in cylindrical co-ordinates (r, θ, z). Any geometry of machine operating in any manner can be represented as

$$\phi_r = \sum_{j=1}^{\infty} \mathbb{A}_j \mathcal{H}_m(kr) \mathrm{e}^{im\theta} \mathcal{Z}_j(z)\, \mathrm{e}^{\mathrm{i}\alpha t}, \tag{5}$$

where $\mathbb{A}_j$ is a complex amplitude, $\mathcal{H}_m$ are Hankel functions, $\mathcal{Z}_j(z)$ is the classical solution to Laplace's equation for the vertical structure of ocean waves, and the index j represents a unique combination of the wavenumber k, where $k = 2\pi/\lambda$, and of the azimuthal wavenumber m, with λ being wavelength. In the literature, e.g. [9], a set of functions in the form of (5) is used not only to represent ϕ_r but also ϕ_i and ϕ_d. The value of ϕ_r created by a neighbouring machine some distance $r = d$ away represents its driving applied to the local machine, and vice versa.

To account for multiple scattering, the physical displacement δ in (1) is replaced with a new 'self-consistent' variable x_i, where the index i identifies each machine. The self-consistent approach [21, 13] eliminates the need for an infinitely nested set of sums. Unfortunately, x is an unphysical variable – it represents the value of the physical variable δ after all multiple-scattering interactions have occurred. However, if carefully interpreted, the self-consistent approach is an appropriate technique [13].

For a pair of machines, the coupled model, using the same scalings as for (1), is

$$\begin{aligned} \ddot{x}_1 + \gamma\ddot{x}_2(t-\tau) + 2\zeta\dot{x}_1 + x_1 + \varepsilon\mathcal{N}(x_1) &= \zeta \mathrm{e}^{\mathrm{i}\alpha t}, \\ \ddot{x}_2 + \gamma\ddot{x}_1(t-\tau) + 2\zeta\dot{x}_2 + x_2 + \varepsilon\mathcal{N}(x_2) &= \zeta \mathrm{e}^{\mathrm{i}\alpha t}, \end{aligned} \tag{6}$$

in which γ represents the strength of the coupling, and τ is the time delay owing to propagation from one machine to another. The values of γ and τ are determined from ϕ_r in (5) [7] and are the magnitude and phase of the complex number usually called the 'hydrodynamic coefficient' in WEC literature. The nonlinear term is given by $\mathcal{N}(x_i) = x_i\ddot{x}_i + C_f|\dot{x}_i|^{3/4}\dot{x}_i$, as in (1).

The results are shown in figure 2, zoomed-in around the resonance. Six machine radii separate the machine centres, corresponding to about 0.34λ. The coupled linear machines (thick grey line) show a slightly higher response than the single linear machine, as expected for these parameters [4], and also a lower resonance frequency. Now, comparison of the coupled nonlinear machines (thick black line) with the coupled linear machines (thick grey line) shows a slight reduction in resonant frequency and a drop in response amplitude of about 10%. Since power is proportional to the square of amplitude, this is a significant drop in power from the prediction of linear theory.

Conclusions

Nonlinearities in an oscillating water column were studied using a simple ordinary differential equation representation of the dynamics of a single OWC. Two nonlinearities were considered, the displacement-dependent inertia and the parameterised turbulent losses. The nonlinear inertia was found to be more significant, reducing the amplitude at resonance by about 15%, but even so only under conditions representing exceptionally large incident waves that would not be expected under normal conditions. Under realistic conditions, a reduction of resonant amplitude of only about 8% occurs, and the time dependence appears virtually sinusoidal. A calculation with two coupled nonlinear machines showed a reduction in resonant frequency beyond that expected if their behaviour were linear. There is also an approximately 10% reduction in amplitude, relative to the pair of coupled linear machines. Based on these results, it is possible that the dynamics of coupled OWCs could be modelled assuming linear machine dynamics, but care should be taken to check the nonlinear amplitude near resonance.

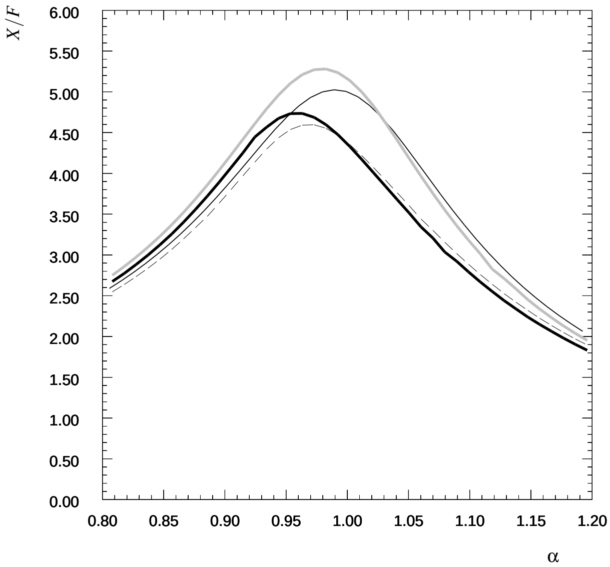

Figure 2: Amplitudes of nonlinear response X/F of water inside two coupled, nonlinear OWCs (thick black line) as a function of dimensionless forcing frequency, considering both nonlinearities as in figure 1. Thin line: classical response function for a linear oscillator. Dashed line: response of a single, nonlinear OWC. Thick grey line: response of the two coupled OWCs assuming linear behaviour. Damping ratio $\zeta_\mu = 0.1$, $\varepsilon = 0.846$.

The present nonlinear results assume the linear damping due to all factors, including the power take-off, is a constant. In reality, the linear radiation damping is frequency-dependent. The present calculations are therefore of greatest validity near resonance. Furthermore, the nonlinear damping term represented turbulent losses on the internal walls bounding the water column, but not losses due to entrance vortex formation. Thus, the present work must be considered preliminary and should be revisited to make a more comprehensive determination of the nonlinear behaviour.

Acknowledgements

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References

[1] Akhavan, R., Kamm, R., and Shapiro, A., An investigation of transition to turbulence in bounded oscillatory Stokes flows. part 1. experiments. *J. Fluid Mech.*, **225**(1), 1991, 395–422.

[2] S. P. Beeby, M. J. Tudor, N. M. White, Energy harvesting vibration sources for microsystems applications, *Meas. Sci. Technol.*, **17**, 2006, R175–R195.

[3] Behrens, S., Hayward, J., Hemer, M., and Osman, P., Assessing the wave energy converter potential for Australian coastal regions. *Renew. Energy*, **43**, 2012, 210–217.

[4] Budal, K., Theory for absorption of wave power by a system of interacting bodies. *J. Ship Res.*, **21**(4), 1977, 248–253.

[5] European Marine Energy Centre, Wave Developers, http://www.emec.org.uk/marineenergy/wave-developers/, Retrieved 12 Aug, 2016.

[6] de Andrés, A. D., Guanche, R., Meneses, L., Vidal, C., Losada, I., Factors that influence array layout on wave energy farms, *Ocean Eng.* **82**, 2014, 32–41.

[7] De Chowdhury, S. and Manasseh, R., Behaviour of eigenmodes of an array of oscillating water column devices. *Wave Motion* (under review), 2016.

[8] Evans, D., Wave-power absorption by systems of oscillating surface pressure distributions. *J. Fluid Mech.*, **114**, 1992, 481–499.

[9] Evans, D. and Porter, R. (1997). Efficient calculation of hydrodynamic properties of OWC-type devices. *Journal of Offshore Mechanics and Arctic Engineering*, 119:210–218.

[10] Falcão, A. F. O., Wave energy utilization: A review of the technologies, Renew. Sustain. Energy Rev. **14**, 2010, 899–918.

[11] Falnes, J. and Budel, K., Wave-power absorption by parallel rows of interacting oscillating bodies. *Appl. Ocean Res.*, **4**(4), 1982, 194–207.

[12] Falnes, J. and McIver, P., Surface wave interactions with systems of oscillating bodies and pressure distributions. *Appl. Ocean Res.*, **7**(4), 1985, 225–234.

[13] Feuillade, C., Scattering from collective modes of air bubbles in water and the physical mechanism of superresonances. *J. Acous. Soc. Am.*, **98**, 1995, 1178–1190.

[14] Hasan, M. K. and Manasseh, R., Dissipation in oscillating water columns. 19th Australasian Fluid Mechanics Conference, Melbourne, Australia, 8-11 December, 2014, 365.

[15] Hasan, M. K., Manasseh, R., Leontini, J. S., Analytic representations of dissipation in Oscillating Water Columns. *Appl. Math. Model.* (under review), 2016.

[16] Heath, T., A review of oscillating water columns. *Phil. Trans. Roy. Soc. A: Math., Phys. & Eng. Sci.*, **370**(1959), 2012, 235–245.

[17] Illesinghe, S. and Hasan, M. K., Manasseh, R., Multiphase dynamics of oscillating-water-column ocean wave energy converters. *18th Australasian Fluid Mechanics Conference, Launceston, Australia, 3-7 December*, 2012, 371.

[18] Meylan, M. H. and Fitzgerald, C. The singularity expansion method and near-trapping of linear water waves. *J. Fluid Mech.*, **755**, 2014, 230–250.

[19] Nader, J.-R., Zhu, S.-P., Cooper, P., and Stappenbelt, B., A finite-element study of the efficiency of arrays of oscillating water column wave energy converters. *Ocean Eng.*, **43**, 2012, 72–81.

[20] Sarpkaya, T. and Storm, M., In-line force on a cylinder translating in oscillatory flow. *Appl. Ocean Res.*, **7**(4), 1985, 188–196.

[21] Tolstoy, I., Superresonant systems of scatterers. I. *J. Acoust. Soc. Am.*, **80**(1), 1986, 282–294.

[22] Wolgamot, H., Effects of second-order hydrodynamics on efficiency of a wave energy array. *11th European Wave and Tidal Energy Conference*, Nantes, France, 6–11 September, 2015.